

The Weinberg Propagators

By

Valeri V. Dvoeglazov¹

Escuela de Física, Universidad Autónoma de Zacatecas
Antonio Dovalí Jaime s/n, Zacatecas 98068, Zac., México
Internet address: VALERI@CANTERA.REDUAZ.MX

Abstract. An analog of the $j = 1/2$ Feynman-Dyson propagator is presented in the framework of the $j = 1$ Weinberg's theory. The basis for this construction is the concept of the Weinberg field as a system of four field functions differing by parity and by dual transformations.

Accordingly to the Feynman-Dyson-Stueckelberg ideas, a causal propagator has to be constructed by using the formula (e. g., ref. [1, p.91])

$$S_F(x_2, x_1) = \int \frac{d^3k}{(2\pi)^3} \frac{m}{E_k} \left[\theta(t_2 - t_1) a u^\sigma(k) \otimes \bar{u}^\sigma(k) e^{-ikx} + \theta(t_1 - t_2) b v^\sigma(k) \otimes \bar{v}^\sigma(k) e^{ikx} \right] , \quad (1)$$

$x = x_2 - x_1$. In the $j = 1/2$ Dirac theory it results to

$$S_F(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\hat{k} + m}{k^2 - m^2 + i\epsilon} , \quad (2)$$

provided that the constant a and b are determined by imposing

$$(i\hat{\partial}_2 - m)S_F(x_2, x_1) = \delta^{(4)}(x_2 - x_1) , \quad (3)$$

¹On leave of absence from *Dept. Theor. & Nucl. Phys., Saratov State University, Astrakhanskaya ul., 83, Saratov RUSSIA*. Internet address: dvoeglazov@main1.jinr.dubna.su

namely, $a = -b = 1/i$.

However, attempts to construct the covariant propagator in this way have failed in the framework of the Weinberg theory, ref. [2], which is a generalization of the Dirac's ideas to higher spins. For instance, on the page B1324 of ref. [2] Weinberg writes:

“Unfortunately, the propagator arising from Wick's theorem is NOT equal to the covariant propagator except for $j = 0$ and $j = 1/2$. The trouble is that the derivatives act on the $\epsilon(x) = \theta(x) - \theta(-x)$ in $\Delta^C(x)$ as well as on the functions² Δ and Δ_1 . This gives rise to extra terms proportional to equal-time δ functions and their derivatives. . . The cure is well known: . . . compute the vertex factors using only the original covariant part of the Hamiltonian \mathcal{H} ; do not use the Wick propagator for internal lines; instead use the covariant propagator.

The propagator, recently proposed in refs. [4, 5] (see also ref. [3]), is the causal propagator. However, the old problem remains: the Feynman-Dyson propagator is not the Green's function of the Weinberg equation. As mentioned, the covariant propagator proposed by Weinberg propagates kinematically spurious solutions [5]. . . The aim of my paper is to consider the problem of constructing the propagator in the framework of the model given in [6, 7]. The concept of the Weinberg field “doubles” has been proposed there. It is based on the equivalence between a Weinberg field and an antisymmetric tensor field, ref. [6], which can be described by $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$. These field functions may be used to form a parity doublet. An essential ingredient of the consideration of ref. [7] is the idea of combining the Lorentz and the dual transformation. This idea, in fact, has been proposed in refs. [3, 4]. An example of such combining is a Bargmann-Wightman-Wigner-type quantum field theory, ref. [3b].

The set of four equations has been proposed in ref. [6]. For the functions $\psi_1^{(1)}$ and $\psi_2^{(1)}$, connected with the first one by the dual (chiral, γ_5) transformation, the equations are

$$(\gamma_{\mu\nu} p_\mu p_\nu + m^2) \psi_1^{(1)} = 0 \quad , \quad (4)$$

$$(\gamma_{\mu\nu} p_\mu p_\nu - m^2) \psi_2^{(1)} = 0 \quad . \quad (5)$$

For the field functions connected with $\psi_1^{(1)}$ and $\psi_2^{(1)}$ by $\gamma_5 \gamma_{44}$ transformations the set of equations is written:

$$[\tilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2] \psi_1^{(2)} = 0 \quad , \quad (6)$$

$$[\tilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2] \psi_2^{(2)} = 0 \quad , \quad (7)$$

where $\tilde{\gamma}_{\mu\nu} = \gamma_{44} \gamma_{\mu\nu} \gamma_{44}$ is connected with the Barut-Muzinich-Williams $j = 1$ matrices [8].

In the cited papers I have used the plane-wave expansion

$$\psi_1(x) = \sum_{\sigma} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{m \sqrt{2E_p}} \left[u_1^{\sigma}(\vec{p}) a_{\sigma}(\vec{p}) e^{ipx} + v_1^{\sigma}(\vec{p}) b_{\sigma}^{\dagger}(\vec{p}) e^{-ipx} \right] \quad , \quad (8)$$

²In the cited paper $\Delta_1(x) \equiv i [\Delta_+(x) + \Delta_+(-x)]$ and $\Delta(x) \equiv \Delta_+(x) - \Delta_+(-x)$ have been used. $i\Delta_+(x) \equiv \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_p} \exp(ipx)$ is a particle Green's function.

$$\psi_2(x) = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[u_2^{\sigma}(\vec{p}) c_{\sigma}(\vec{p}) e^{ipx} + v_2^{\sigma}(\vec{p}) d_{\sigma}^{\dagger}(\vec{p}) e^{-ipx} \right] , \quad (9)$$

where $E_p = \sqrt{\vec{p}^2 + m^2}$, in order to prove that one can describe a $j = 1$ quantum particle with transversal components in the framework of the Weinberg and/or of the antisymmetric tensor theory.

The corresponding bispinors in the momentum space coincide with the Tucker-Hammer ones within a normalization.³ Their explicit form is

$$u_1^{\sigma(1)}(\vec{p}) = v_1^{\sigma(1)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m + (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \\ \left[m - (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \end{pmatrix} , \quad (10)$$

and

$$u_2^{\sigma(1)}(\vec{p}) = v_2^{\sigma(1)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m + (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \\ \left[-m + (\vec{J}\vec{p}) - \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \end{pmatrix} . \quad (11)$$

Thus, $u_2^{(1)}(\vec{p}) = \gamma_5 u_1^{(1)}(\vec{p})$ and $\bar{u}_2^{(1)}(\vec{p}) = -\bar{u}_1^{(1)}(\vec{p})\gamma_5$.

Bispinors

$$u_1^{\sigma(2)}(\vec{p}) = v_1^{\sigma(2)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[m - (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \\ \left[-m - (\vec{J}\vec{p}) - \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \end{pmatrix} , \quad (12)$$

$$u_2^{\sigma(2)}(\vec{p}) = v_2^{\sigma(2)}(\vec{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[-m + (\vec{J}\vec{p}) - \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \\ \left[-m - (\vec{J}\vec{p}) - \frac{(\vec{J}\vec{p})^2}{(E+m)} \right] \xi_{\sigma} \end{pmatrix} \quad (13)$$

satisfy Eqs. (6) and (7) written in the momentum space. Thus, $u_1^{(2)}(\vec{p}) = \gamma_5 \gamma_{44} u_1^{(1)}(\vec{p})$, $\bar{u}_1^{(2)} = \bar{u}_1^{(1)} \gamma_5 \gamma_{44}$, $u_2^{(2)}(\vec{p}) = \gamma_5 \gamma_{44} \gamma_5 u_1^{(1)}(\vec{p})$ and $\bar{u}_2^{(2)}(\vec{p}) = -\bar{u}_1^{(1)} \gamma_{44}$.

Let me check, if the sum of four equations ($x = x_2 - x_1$)

$$\begin{aligned} & \left[\gamma_{\mu\nu} \partial_{\mu} \partial_{\nu} - m^2 \right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1) a \ u_1^{\sigma(1)}(p) \otimes \bar{u}_1^{\sigma(1)}(p) e^{ipx} + \right. \\ & \quad \left. + \theta(t_1 - t_2) b \ v_1^{\sigma(1)}(p) \otimes \bar{v}_1^{\sigma(1)}(p) e^{-ipx} \right] + \\ & + \left[\gamma_{\mu\nu} \partial_{\mu} \partial_{\nu} + m^2 \right] \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1) a \ u_2^{\sigma(1)}(p) \otimes \bar{u}_2^{\sigma(1)}(p) e^{ipx} + \right. \\ & \quad \left. + \theta(t_1 - t_2) b \ v_2^{\sigma(1)}(p) \otimes \bar{v}_2^{\sigma(1)}(p) e^{-ipx} \right] + \end{aligned}$$

³They also coincide with the bispinors of Ahluwalia *et al.*, ref. [4], within a unitary transformation.

$$\begin{aligned}
& + \left[\tilde{\gamma}_{\mu\nu} \partial_\mu \partial_\nu + m^2 \right] \int \frac{d^3 p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1) a u_1^{\sigma(2)}(p) \otimes \bar{u}_1^{\sigma(2)}(p) e^{ipx} + \right. \\
& \quad \left. + \theta(t_1 - t_2) b v_1^{\sigma(2)}(p) \otimes \bar{v}_1^{\sigma(2)}(p) e^{-ipx} \right] + \\
& + \left[\tilde{\gamma}_{\mu\nu} \partial_\mu \partial_\nu - m^2 \right] \int \frac{d^3 p}{(2\pi)^3 2E_p} \left[\theta(t_2 - t_1) a u_2^{\sigma(2)}(p) \otimes \bar{u}_2^{\sigma(2)}(p) e^{ipx} + \right. \\
& \quad \left. + \theta(t_1 - t_2) b v_2^{\sigma(2)}(p) \otimes \bar{v}_2^{\sigma(2)}(p) e^{-ipx} \right] = \delta^{(4)}(x_2 - x_1) \quad (14)
\end{aligned}$$

can be satisfied by the definite choice of a and b . The relation $u_i(p) = v_i(p)$ for bispinors in the momentum space had been also used in ref. [7, 9]. In the process of calculations I assume that the set of “Pauli spinors”⁴ is the complete set and it is normalized to $\delta_{\sigma\sigma'}$.

The simple calculations yield

$$\begin{aligned}
& \partial_\mu \partial_\nu \left[a \theta(t_2 - t_1) e^{ip(x_2 - x_1)} + b \theta(t_1 - t_2) e^{-ip(x_2 - x_1)} \right] = \\
& = -[a p_\mu p_\nu \theta(t_2 - t_1) \exp[ip(x_2 - x_1)] + b p_\mu p_\nu \theta(t_1 - t_2) \exp[-ip(x_2 - x_1)]] + \\
& + a [-\delta_{\mu 4} \delta_{\nu 4} \delta'(t_2 - t_1) + i(p_\mu \delta_{\nu 4} + p_\nu \delta_{\mu 4}) \delta(t_2 - t_1)] \exp[i\vec{p}(\vec{x}_2 - \vec{x}_1)] + \\
& + b [\delta_{\mu 4} \delta_{\nu 4} \delta'(t_2 - t_1) + i(p_\mu \delta_{\nu 4} + p_\nu \delta_{\mu 4}) \delta(t_2 - t_1)] \exp[-i\vec{p}(\vec{x}_2 - \vec{x}_1)] \quad ; \quad (15)
\end{aligned}$$

and

$$u_1^{(1)} \bar{u}_1^{(1)} = \frac{1}{2} \begin{pmatrix} m^2 & S_p \otimes S_p \\ \bar{S}_p \otimes \bar{S}_p & m^2 \end{pmatrix} \quad , \quad u_2^{(1)} \bar{u}_2^{(1)} = \frac{1}{2} \begin{pmatrix} -m^2 & S_p \otimes S_p \\ \bar{S}_p \otimes \bar{S}_p & -m^2 \end{pmatrix} \quad , \quad (16)$$

$$u_1^{(2)} \bar{u}_1^{(2)} = \frac{1}{2} \begin{pmatrix} -m^2 & \bar{S}_p \otimes \bar{S}_p \\ S_p \otimes S_p & -m^2 \end{pmatrix} \quad , \quad u_2^{(2)} \bar{u}_2^{(2)} = \frac{1}{2} \begin{pmatrix} m^2 & \bar{S}_p \otimes \bar{S}_p \\ S_p \otimes S_p & m^2 \end{pmatrix} \quad , \quad (17)$$

where

$$S_p = m + (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{E + m} \quad , \quad (18)$$

$$\bar{S}_p = m - (\vec{J}\vec{p}) + \frac{(\vec{J}\vec{p})^2}{E + m} \quad . \quad (19)$$

Due to

$$\begin{aligned}
& [E_p - (\vec{J}\vec{p})] S_p \otimes S_p = m^2 [E_p + (\vec{J}\vec{p})] \quad , \\
& [E_p + (\vec{J}\vec{p})] \bar{S}_p \otimes \bar{S}_p = m^2 [E_p - (\vec{J}\vec{p})] \quad .
\end{aligned}$$

one can conclude: the generalization of the notion of causal propagators is admitted by using “Wick’s formula” for the time-ordered particle operators provided that $a = b = 1/4im^2$. It is necessary to consider all four equations, Eqs. (4)-(7).

The $j = 1$ analogues of the formula (2) for the Weinberg propagators follow from the formula (3.6) of ref. [4] immediately:⁵

$$S_F^{(1)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} \left[\gamma_{\mu\nu} p_\mu p_\nu - m^2 \right] \quad , \quad (20)$$

⁴I mean their analogues in the $(1, 0)$ or $(0, 1)$ spaces.

⁵Please do not forget that I use the Euclidean metric as in my previous papers.

$$S_F^{(2)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} [\gamma_{\mu\nu} p_\mu p_\nu + m^2] \quad , \quad (21)$$

$$S_F^{(3)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2] \quad , \quad (22)$$

$$S_F^{(4)}(p) \sim -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2] \quad . \quad (23)$$

It is interesting to note that the causal propagator consisting of four terms, four parts, four propagators has been met earlier. Namely, in the bound state theory. You may compare the propagators which is above with the Green's function for the two-fermion system, ref. [10, 11]:⁶

$$G_0 = i(2\pi)^4 \delta(p - q) S_1(p_1) S_2(p_2) \quad , \quad (24)$$

$$S_i = - \left[\Lambda_i^+(\vec{p}_i)(p_{0i} - E_{ip} + i\epsilon)^{-1} + \Lambda_i^-(\vec{p}_i)(p_{0i} + E_{ip} + i\epsilon)^{-1} \right] \gamma_{i0} \quad , \quad (25)$$

Λ_i^\pm are the projection operators.

We should use the obtained set of Weinberg propagators (20,21,22,23) in perturbation calculus of scattering amplitudes. In ref. [13] the amplitude for the interaction of two $2(2j + 1)$ bosons has been obtained on the basis of the use of one field only and it is obviously incomplete, see also ref. [9]. But, it is interesting that the spin structure has proved there not to be changed regardless we consider the two-Dirac-fermion interaction or the two-Weinberg($j = 1$)-boson interaction. However, the denominator slightly differs ($1/\vec{\Delta}^2 \rightarrow 1/2m(\Delta_0 - m)$) in the cited papers [13] from the fermion-fermion case. More accurate consideration of the fermion-boson and boson-boson interactions in the framework of the Weinberg theory is in progress.

The conclusions are: one can construct an analog of the Feynman-Dyson propagator for the $2(2j + 1)$ model and, hence, a “local” theory provided that the Weinberg states are “quadrupled” ($j = 1$ case).

Acknowledgments. The papers of Dr. D. V. Ahluwalia led me to the ideas presented here and in my previous articles. His answers on my questions were very helpful in realizing the importance of presented topics. I thank him, Dr. I. G. Kaplan and Dr. A. Mondragon for encouragements; Dr. Yu. F. Smirnov for asking the right questions at the right time.

I am grateful to Zacatecas University for professorship.

References

- [1] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*. (McGraw-Hill Book Co. New York, 1980)

⁶For a recent review see ref. [12].

- [2] S. Weinberg, Phys. Rev. B**133** (1964) 1318
- [3] D. V. Ahluwalia and D. J. Ernst, Phys. Lett. B**287** (1992) 18; D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. B**316** (1993) 102
- [4] D. V. Ahluwalia and D. J. Ernst, Int. J. Mod. Phys. E**2** (1993) 397
- [5] D. V. Ahluwalia and D. J. Ernst, Phys. Rev. C**45** (1992) 3010
- [6] V. V. Dvoeglazov, *Mapping between antisymmetric tensor and Weinberg formulations*. Preprint EFUAZ FT-94-05, Zacatecas, August 1994
- [7] V. V. Dvoeglazov, *What particles are described by the Weinberg theory?* Preprint EFUAZ FT-94-06, Zacatecas, August 1994
- [8] A. O. Barut, I. Muzinich and D. Williams, Phys. Rev. **130** (1963) 442
- [9] R. H. Tucker and C. L. Hammer, Phys. Rev. D**3** (1971) 2448
- [10] J. Schwinger, Proc. Nat. Acad. Sci. (USA) **37** (1951) 452, 455; E. E. Salpeter and H. A. Bethe, Phys. Rev. **84** (1951) 1232; A. A. Logunov and A. N. Tavkhelidze, Nuovo Cim. **29** (1963) 380; R. N. Faustov, Nucl. Phys. **75** (1966) 669
- [11] Yu. N. Tyukhtyaev, Teor. Mat. Fiz. **53** (1982) 419 [English translation: Theor. Math. Phys. **53** (1982) 1217]; N. A. Boikova *et al.*, Teor. Mat. Fiz. **89** (1991) 228 [English translation: Theor. Math. Phys. **89** (1991) 1174]
- [12] V. V. Dvoeglazov, Yu. N. Tyukhtyaev and R. N. Faustov, Fiz. Elem. Chast. At. Yadra **25** (1994) 144 [English translation: Phys. Part. Nucl. **25** (1994) 58]
- [13] V. V. Dvoeglazov and N. B. Skachkov, JINR Communications P2-87-882, Dubna:JINR, 1987 [in Russian]; V. V. Dvoeglazov, Yu. N. Tyukhtyaev and S. V. Khudyakov, Izvestiya VUZov:fiz. **37**, No. 9 (1994) 110 [English translation: Russ. Phys. J. **37** (1994) 898]; V. V. Dvoeglazov and S. V. Khudyakov, *Gluonium as a bound state of massive gluons described by the Joos-Weinberg wave functions*. Preprint IFUNAM FT-94-35 (hep-ph/9311347), Mexico, 1993